Chapter Twelve

Design for Commodity-by-Industry Interregional Input-Output Models

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In this paper we do not attempt to develop an interregional model. Rather, we propose a mathematical structure of computational framework with which a family of interregional input-output models can be constructed.

This family of interregional input-output models encompasses the well-known class of static models that are used to analyse the propagation of demand throughout an economic system disaggregated both regionally and industrially. Among the most familiar models of this class are the national-intranational model of Leontief [8], the Leontief-Strout gravity model [9], and those associated with CheneFY [2] , Isard [7], and Moses [11] .

In our view, it is useful to consider an interregional model as a disaggregation of a national model, rather than as a formal linking of individual regional models. Many of the data required to construct input-output accounts are available at the national level, but not at the subnationallevel. This is particularly true of interregional trade data and some elements on the income side of the national income and expenditure accounts. The approach of disaggregation does not require estimation of the complete network of interregional trade flows nor allocation of all income to each region. The national input-output accounting framework, which serves as the starting point in the development of the interregional input-output model framework proposed in this paper, is the commodity-by-industry accounting framework associated with Statistics Canada input-output models [3; 12]. The essence of the accounting framework is the recognition of two spaces: an institutional space where institutions are grouped into industries, households, governments, and a foreign sector; and a transaction or commodity space that distinguishes intermediate flows or flows of produced goods and primary flows or factor inputs. The accounts show the production of commodities by industries; they also show foreign imports and the use of both as intermediate inputs

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or objects of final expenditure. Each industry may produce more than one commodity and each commodity may be produced by more than one industry.

Figure 12-1 represents such a set of national input-output accounts for a. fictitious economy that consists of three industries, four commodities, two factors, and four final demand sectors. The accounts "balance" in two ways-the total supply of commodities is equal to the total disposition of commodities, and the total industry outputs are equal to the total industry inputs.

Mathematical Structure

From this commodity-by-industry accounting framework, the parameters for an input-output model under the assumptions of industry technology, fixed domestic market shares, and fixed import shares can be calculated [4; 12]. The mod~l consists of three basic equations:

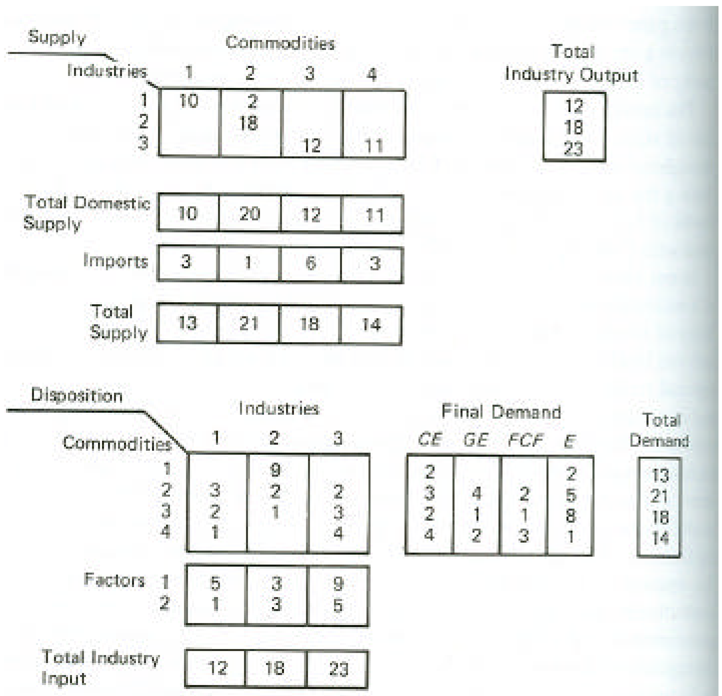

Figure 12-1. A Set of National Input-Output Accounts. *CE* and *GE* are, respectively, consumer and government expenditures; *FCF* is fixed capital formation; and *X* is exports.

$$Q + M = AX + Y + E \tag{12-1}$$

$$X = DQ \tag{12-2}$$

$$M = \hat{\mu}(AX + Y) \tag{12-3}$$

where

Q = vector of domestic output by commodities

M = vector of imports by commodities

A = matrix of input coefficients

X = vector of domestic outputs by industries

Y = vector of domestic final demand by commodities

E = vector of exports by commodities

D = matrix of domestic market share coefficients

$\hat{\mu}$ = diagonal matrix of import share coefficients

The first equation states that the total supply of commodities, $Q + M$, must be equal to their total disposition which consists of intermediate demand, AX, and final demand, $Y + E$. The A coefficients reflect the assumption of industry technology and are calculated by dividing the intermediate inputs of commodities by the appropriate industry outputs, that is,

$$A = U\hat{X}^{-1}$$

where U is the matrix of inputs into industries.

The second equation states that the output of domestically produced commodities is allocated among industries according to the market share coefficients D. They are obtained by dividing each element of the output matrix by the appropriate elements of total domestic output, that is,

$$D = V\hat{Q}^{-1}$$

where V is the matrix of outputs by commodities and industries.

The third equation states that imports are a share of total domestic demand, $AX + Y$. The import share coefficients are obtained by dividing imports by total domestic demand, that is,

$$\mu = (\hat{Z})^{-1}M$$

where $Z = AX + Y$.

These three equations form an input-output model that can be solved for .X, *Q*, and *M* in terms of *Y* and *E*. The solution for *X* is given by:

$$X = [I - D(I - U)B]^{-1}D(I - U)\, Y + DE \tag{12-4}$$

A computational framework or computer algorithm has been developed for solving models of this mathematical structure. The most distinguishing character. istic of the computer algorithm is that it does not calculate or make use of an inverse or impact table such as that set out in equation (12-4). Rather, the computer system is used to calculate a specific solution. (No general solution is obtained.) This approach was chosen for reasons of computational efficiency and flexibility in changing parameter arrays. The large arrays of parameters required by the model-namely, the input coefficients and the domestic market share coefficients-are stored and manipulated in "compact" form. The main feature of the compact form is that only the non-zero elements in the arrays are handled. A matrix is represented by three vectors: a vector of the nonzero elements taken row by row; a vector whose elements are the column identification of the cor. responding elements in the first vector; and a vector whose elements are the number of elements in each row of the matrix. The number of elements in each of the first two vectors is equal to the number of nonzero elements in the original matrix, and the number of elements in the third vector is equal to the num. ber of rows in the original matrix. The expression of matrices in compact form is significant for input-output calculations because the coefficient arrays are extremely sparse. Since inverse matrices are by nature not sparse, it is more efficient to store sparse parameter matrices and to calculate the single solutions that use them rather than to store and manipulate inverse matrices.

It is to be noted, as well, that the use of a single-solution procedure avoids the necessity of recalculating an inverse matrix each time the coefficient arrays are changed. Accordingly, the single-solution system is convenient for analysing the impact of changes in the *structure* of the economy.

The solution of the model is achieved by means of the iterative process set out in Figure 12-2. Block 1 of the figure allocates final demand among direct imports, *M*, and domestic output by commodities, *Q*.

Block 3 allocates the domestic outputs by commodities, Q, among industries according to the domestic market share coefficients. Block 6 then calculates the indirect domestic commodity production required as inputs in order to satisfy the additional demands. These additional demands, ~*Q*, for domestic output by commodities are allocated among industries in Block 3.

The system iterates over Block 3 to Block 6, inclusive. At each iteration, the increments to industry outputs, M, are accumulated in the *X* vector. This occurs in Block 4. Intuitively, convergence is assured, insomuch as the increments to outputs by industries diminish from iteration to iteration because of the leakages

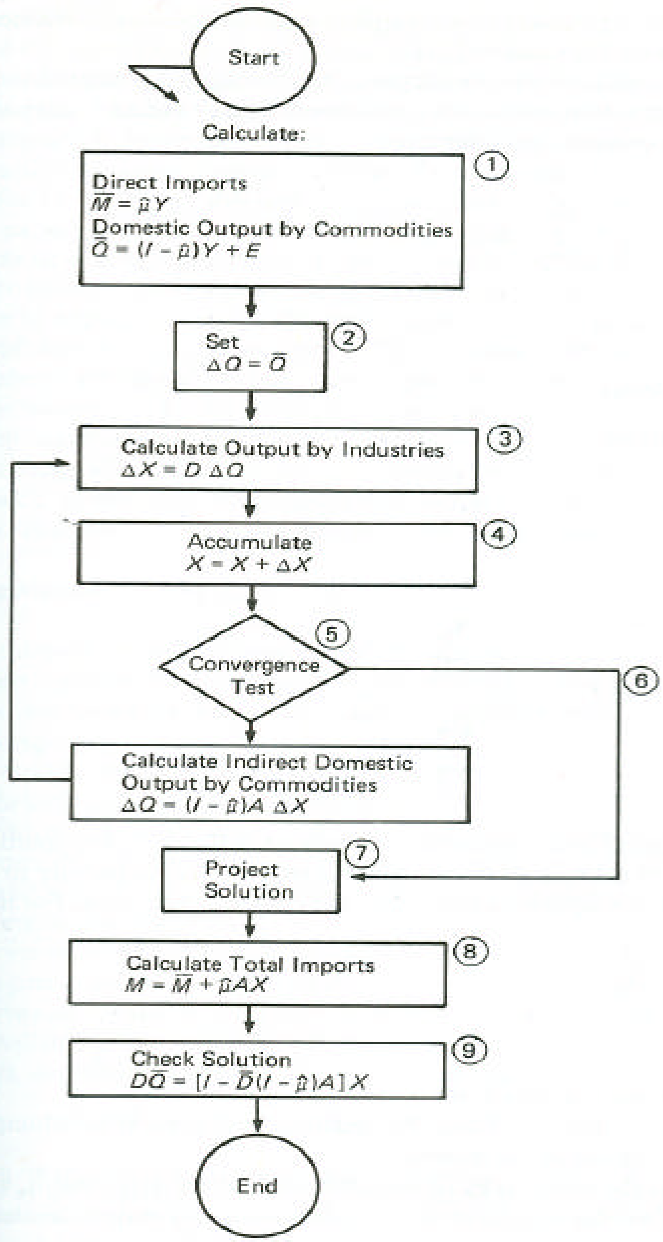

Figure 12–2. The Solution of the Model.

to imports and primary inputs that take place at each iteration. A mathematic~ proof of convergence is available **[12].**

The iterations proceed until a measure of the average increment in industry outputs is approximately equal to the increment in each industry output. More precisely, when a prespecified tolerance,

$$\epsilon \geqslant \sum_{j=1}^{n} \frac{e_j}{1-e_j} \Delta x_{ij} \div \sum_{j=1}^{n} \Delta x_{oj}$$

where

$j = 1, \ldots, n$ industries

$i = 1, \ldots, k$ iterations

and

$$x_j = r_j - \bar{r}$$

where

$$r_j = \frac{\Delta x_{ij}}{\Delta x_{i-1,j}} \qquad \text{and} \qquad r = \frac{\sum_{j=1}^{n} \Delta x_{ij}}{\sum_{j=1}^{n} \Delta x_{i-1,j}}$$

convergence is assured and a solution is calculated in Block 7. The solution for industry outputs is the sum of the increments to industry outputs up to the iteration in which the tolerance is met plus a projected increment. For industry

$$x_j = \sum_{i=1}^{k} \Delta x_{ij} + \frac{r_j}{1-r} \Delta x_{kj}$$

where k is the iteration in which the tolerance is met.

Given industry outputs, X, Block 8 calculates total imports by adding direct requirements to indirect requirements.

Block 9 checks the solution by running the system in reverse, that is, by calcu. lating domestic final demand from the calculated industry output levels.

The mathematical structure and computational framework that have been described thus far have two features that are pertinent for interregional models.

The commodity-by-industry accounting framework facilitates the separation of supply or marketing relationships from input or technological relationships. The mathematical structure we have chosen may be interpreted as follows: at each iteration, demand is pooled and then allocated among sources of supply.

This notion of "pooling" wi1llead to the definition of subnational "pools" and separate patterns of supply for each pool.

The use of compact parameter arrays and a single-solution algorithm makes it feasible to handle efficiently very large matrices, recognizing that these matrices are apt to be extremely sparse. The limiting characteristic from the computational point of view is not the dimensions of the matrices, but the number of nonzero numbers. This feature is important especially for interregional models, in which the dimensions of the coefficient matrix are usually the product of the number of industries and the number of regions.

The strategy adopted for introducing the regional dimension into such a framework is simply that of disaggregation within the existing mathematical structure. Thus we proceed by redefining the industry and commodity spaces.

Disaggregation in Industry Space

Because the industry space in the national input-output accounts is institutional, the "industries" may be redefined to be industries in regions. Accordingly, the mix of products and the pattern of inputs for each industry may vary from region to region.

Furthermore, the regional disaggregation of industries may be *selective.*

Certain industries may be designated as "national" industries and therefore need not be disaggregated at all. For industries, such as transportation and communications, that involve a national, network and for which the assignment of outputs and inputs to a particular region is at best arbitrary, the "national" industry concept is appropriate.l

There are many industries for which only selected inputs and outputs can be given a regional dimension. For instance, in industries subject to an establishment based surveyor census, information on shipments, raw materials, and labour is usually available at the regional level; but information on service inputs, overhead costs, depreciation, and profits is not available. Where only partial regional

1. Even for national industries, primary inputs, such as labour, can be regionally disaggregated by introducing new rows of primary inputs. This is necessary for household income if the model is to be closed with respect to consumer expenditures.

ization is possible, the dummy industry technique may be used to allocate in total the flows for which there is no regional information.

Figure 12-3 is the national accounting framework depicted in Figure 12-1 after disaggregation to distinguish by region.

Industry 1 is a national industry, and therefore remains unchanged in Figure 12-3. Industry 2 has been completely regionalized, thus becoming industries 2.1 and 2.2 in Figure 12-3, where the digit to the right of the decimal specifies the region. Industry 3 has been only partially regionalized in that inputs of commod. ities 3 and 4 and factor 2 could not be regionalized. Therefore, a dummy indus. try, 3.d, has been set up, which produces commodity 5. Commodity 5 is then purchased by industries 3.1 and 3.2.

It is to be noted that the final demand is the same in Figure 12-1 and Figure 12-3, and that there is no direct accounting for interregional trade flows.

Supply: Industries \ Commodities	1	2	3	4	5	Total Industry Output
1	10	2				12
2.1		8				8
2.2		10				10
3.1			12	1		13
3.2				10		10
3.d					12	12
Foreign Imports	3	1	6	3		
Total Supply	13	21	18	14	12	

Disposition: Commodities \ Industries in Regions	1	2.1	2.2	3.1	3.2	3.d	Final Demand: *CE*	*GE*	*FCF*	*E*	Total Demand
1		4	5				2			2	13
2	3	1	1	1	1		3	4	2	5	21
3	2	1				3	2	1	1	8	18
4	1					4	4	2	3	1	14
5				7	5						12
Factors 1	5	1	1	5	4						
Factors 2	1	1	2			5					
Total Industry Input	12	8	10	13	10	12					

Figure 12-3. National Input-Output Accounts with Regional Disaggregation in Industry Space.

The parameters for the model outlined above could be estimated from this new set of accounts. In this case, the market share coefficients serve to allocate demand to regions as well as industries. However, demand is pooled at the national level; therefore, demand originating in one region is met by the same pattern of supply as demand for the same commodity originating in another region.

We turn now to possibilities of disaggregation in commodity space that allow the introduction of alternative models of distribution.

Disaggregation in Commodity Space

Region-specific or customer-specific patterns of supply are introduced by means of disaggregation in commodity space. With the introduction of customer-specific shares, a nUfllber of demand pools may be created for each commodity. Each pool takes the form of a separate row in the disposition matrix. For each row in the disposition matrix there is a corresponding column in the output matrix that gives rise to a pattern of supply. In this way, customer-specific shares are introduced as a simple disaggregation in commodity space, leaving the mathematical structure of the model unchanged.

Let us consider the second commodity of Figure 12-3. Under the assumption of a model formed from the accounts in Figure 12-3, domestic demand, that is, demand originating as intermediate inputs into industries or as domestic final demands, is supplied as follows: 1/16 is imported; that which is supplied domestically is shared among industries 1, 2.1, and 2.2 in the ratio of 2: 8: 10.

Let us assume that we have information that demands originating in industries 2.1 and 3.1 are always supplied by industry 2.1. This information can be introduced into the model by first disaggregating the second row of the disposition matrix, as shown in Figure 12-4, and then by disaggregating the output matrix, as shown in Figure 12-5.

When share coefficients are formed, domestic demand for commodity 2.1 is supplied as follows: 1/14 is imported; that which is supplied domestically is shared among industries 1,2.1, and 2.2 in the ratio of 2:6: 10. Demand for commodity 2.2 is supplied totally by industry 2.1.

Customer-specific shares may be used to depict a range of interregional trading behaviour within the context of the basic model. In fact, each commodity

	Industries										
Commodities	1	2.1	2.2	3.1	3.2	3.d	*CE*	*GE*	*FCF*	*E*	Total
2.1	3		1		1		3	4	2	5	19
2.2		1		1							2

Figure 12-4. Disaggregation of the Second Row of Disposition Matrix.

Industries	Commodities 2.1	Commodities 2.2
1	2	
2.1	6	2
2.2	10	
3.1		
3.2		
3.d		
Foreign Imports	1	
Total	19	2

Figure 12-5. Disaggregation of the Output Matrix.

market may be considered separately. Certain commodities may be designated as local commodities for which production occurs in the region where the demand originates. In this case, demand is pooled by region, and the corresponding market share coefficients direct the demand to industries in the same region. Such commodities as personal services and retail trade may be considered as local commodities.

At the other end of the scale, certain commodities may be treated as national commodities, for which the pattern of supply does not depend upon the region or sector in which the demand originates. This is the behaviour implicit in the basic model. Accordingly, commodities for which this behaviour is held to be true need not be disaggregated at all. This assumption is probably valid for commodities with national distribution networks, where brand name is important and transportation costs are a relatively small portion of the value. A number of consumer durables and semidurables would fall into this category.

For a large number of commodity markets, the distribution patterns are neither "local" nor "national." This is particularly true of many intermediate commodities for which the distribution patterns are influenced by institutional arrangements. For these commodities, customer-specific shares may be used to incorporate information on interregional flows that may be available from transportation statistics or interregional trade surveys. Alternatively, customer-specific shares may be used to incorporate analytic assumptions about trade flows. For example, given sources of supply and location of demand in the base year, customer-specific shares can be calculated in such a way as to minimize transportation costs for particular commodities.

Customer-specific shares may be used selectively and may incorporate fragmentary information on trading patterns. In principle, each nonzero element in the disposition matrix may have its own supply pattern. In the context of an interregional model, it is expected that market shares will be regional-specific for non-national commodities only.

If customer-specific shares are to be introduced in such a way that the baseperiod activity levels can be replicated by a solution of the model using baseperiod final demand, the commodities in the flow matrices should be disaggregated so as to preserve the accounting identities of the system. For each demand pool, the total supply must equal the total disposition of the commodity. In the disposition matrix and the output matrix, the sum of the output or disposition in each $ubset of demand pools must be equal to the corresponding commodity in the account of the basic model.

Implementation

A model within the framework presented in this paper is being implemented by the Structural Analysis Division of Statistics Canada. Work is well advanced on the regionalisation of the 1966 input-output table for Canada. These national input-output tables distinguish more than 650 commodities and 200 industries. It is expected that this degree of detail will be maintained in the regionalized version.

Initially, regionalization means provincialization (ten provinces and two territories). It is recognized, of course, that political boundaries need not coincide with economic boundaries. It is certainly feasible within the context of this model to push regionalization to any level of geographic detail within the limits imposed by the availability of data.

Plans are being made to regionalize the 1971 input-output tables for Canada. Many more commodities are being distinguished in the 1971 tables-as many as 1700. Information on finer levels of geographical detail is being preserved, as well, so that it will be possible to distinguish regions according to a number of criteria.

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